I Semester M.Sc. Degree Examination, January/February 2014
(Semester (NS) Scheme)
MATHEMATICS
M101 : Algebra – I

Time : 3 Hours   Max. Marks : 80

Instructions : *i) Answer **any five** questions choosing at least **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Define a normal subgroup of a group G. Prove that a subgroup N of G is normal subgroup of G if and only if $gNg^{-1} = N$ for each $g \in G$. **6**

   b) Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. Show that for any $n \in N,\ m \in M,\ nm = mn$. **5**

   c) If a cyclic subgroup N of G is normal in G, then show that every subgroup of N is normal in G. **5**

2. a) Define the Kernel of a homomorphism of groups. If $\phi : G \to G'$ is an epimorphism with Kernel K, then prove that ϕ is an isomorphism if and only if K = {e}. **5**

   b) Let ϕ be a homomorphism of G onto $\overline{G}$ with Kernel K, and let $\overline{N}$ be a normal subgroup of $\overline{G}$, $N = \{x \in G : \phi(x) \in \overline{N}\}$. Then prove that $G/N \approx \overline{G}/\overline{N}$. **6**

   c) With usual notations prove that $S_n/A_n \approx \{1, -1\}$. **5**

3. a) Define the inner automorphism of a group G. Prove that the set of all inner automorphisms of a group G is a normal subgroup of the group of all automorphisms of G. **5**

   b) State and prove Cayley's theorem for a finite group. **6**

   c) If G is a finite group and $a \in G$, then prove that $C_a = \dfrac{0(G)}{0(N(a))}$, where N(a) is the normalizer of and C_a is the conjugacy class of a in G. **5**

P.T.O.

4. a) Prove that every group of order p^n, p-being a prime, has non trivial centre. **5**

b) Show that any two p-Sylow subgroups are conjugate to each other. **6**

c) Show that every group of order $11^2.13^2$ is abelian. **5**

PART – B

5. a) Define : **6**

i) integral domain ii) field

prove that every finite integral domain is a field.

b) Define an ideal of a ring. Show that an ideal S of a ring R is an ideal of S +T, where T is any subring of R. **5**

c) Let R be a ring with unity and R not necessarily commutative such that the only left ideals of R are {0} and R. Prove that R is a division ring. **5**

6. a) If R is s commutative ring with unity, then prove that an ideal M of R is maximal if and only if R/M is a field. **6**

b) Define a principal ideal ring. Prove that in a principal ideal ring every non zero prime ideal is maximal. **6**

c) Prove that any two isomorphic integral domains have isomorphic quotient fields. **4**

7. a) Define a Euclidean ring. Prove that an ideal in a Euclidean ring is maximal if and only if it is generated by a prime element. **6**

b) Define g.c.d. of two elements a and b in a ring R. Let R be a Euclidean ring. Then prove that any two elements a and b in R have a g.c.d. Moreover $d = \lambda a + \mu b$, for $\lambda, \mu \in R$, is the g.c.d. of a and b. **6**

c) Let R be a Euclidean ring and $a, b \in R$. Then prove that **4**

i) d(ab) = d(a) if b is a unit in R ii) d(ab) > d(a) if b is not a unit in R.

8. a) Show that F[x] is a Euclidean ring, where F is a field. **6**

b) State and prove Eisenstein criterian for the irreducibility of a polynomial with integer coefficients. **6**

c) Show that the polynomial $1 + x + x^2 + \ldots\ldots + x^{p-1}$, where p is a prime number, is irreducible over the field of rational number. **4**